Final Exam
Physics 319
Note: 5 Problems plus 1 Extra Credit

1. (15 Points) In parts a. and b., identify which of the forces are conservative. For those that are conservative, find the potential function. k and c are constants.
    a. $\vec{F} = k\left(x^2 + y^2\right)\hat{x} + k\left(x^2 - y^2\right)\hat{y}$
    b. $\vec{F} = cxy^2z^2\hat{x} + cx^2yz^2\hat{y} + cx^2y^2z\hat{z}$
    c. Show $\vec{F} = k\dfrac{x\hat{x} + y\hat{y} + z\hat{z}}{x^2 + y^2 + z^2}$ is conservative and find the potential function. For this force field what is a convenient location for the zero of the potential?

2. (20 Points) You are a participant in a precision shooting competition. Your rifle shoots a bullet with muzzle velocity (v_0) of 1000 m/sec and your shot is at a target 1 km away. Assume *g* is 9.8 m/sec^2.
    a. As a zeroth approximation neglect the Coriolis force and centrifugal force from the earth's rotation, and air resistance. If the bull's eye is at height zero, what is the zeroth approximation for the aim point of the shot?
    b. Now add the Coriolis force and assume the target is due east at latitude 40°. How much does the aim point need to be adjusted to hit the bull's eye, and in what direction? What is the adjustment if the shot is due west?
    Extra credit (5 pts): What is the adjusted aim point for a shot at a target due north?

3. (25 Points) Problems on the inertia tensor. Make sure to show your work.
    a. Calculate the inertia tensor for rotations around the center of mass of [1] a uniform cylindrical rod of mass M, length L and radius R (Figure 1), and [2] 4 uniform spherical masses of mass M and radius R whose centers are at the vertices of a square of side L, connected and supported by two massless rods in a cross (Figure 2).
    b. In each of these configurations identify one set of orthogonal principal axes.
    Extra Credit (5 pts): Identify *all* possible sets of orthogonal principal axes.

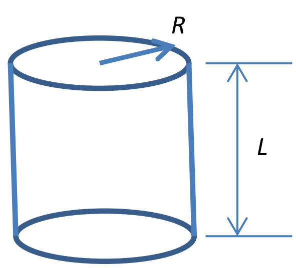

Figure 1

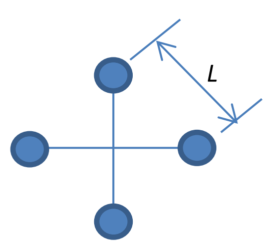

Figure 2

4. (25 Points) A bent wire rotates around a vertical axis with angular frequency Ω at a fixed polar angle θ_0 as shown in Figure 3. A point mass "bead" of mass m can move along the wire with no friction.
   a. What is the Lagrangian for this problem in a suitable coordinate set?
   b. What is the equation of motion?
   c. Show the condition for an equilibrium (circular) orbit is

$$x = \frac{g\cos\theta_0}{\Omega^2 \sin^2\theta_0},$$

   where x is the distance along the wire from the rotation axis to the equilibrium position.
   d. Extra Credit (5 Points) Is the motion stable?

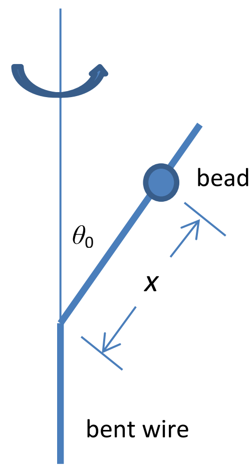

Figure 3. Bead on bent wire problem

5. (15 Points)
   a. Identify the planet with the wrong data recorded in Table 1. The incorrect data is in the period column. Correct the table. Please show your work; there are similar tables on the web.

Planet	Mean orbit radius (AU)	Orbit period (years)
Mercury	0.387	0.241
Earth	1.0	1.0
Mars	1.52	2.18
Jupiter	5.20	11.86
Saturn	9.54	29.46

Table 1: Mean orbit radius and orbit periods for several planets

   b. Given the orbit of Mars has eccentricity 0.0934 and semi-major axis a 2.28×10^8 km, what are the perihelion and aphelion distances in its orbit around the sun?

6. (Extra Credit 15 pts.) A pendulum is formed from a uniform spherical mass of mass M and radius R attached by a massless string so the distance from the fixed point to the center of mass of the sphere is L. The small amplitude oscillation angular frequency is shifted away from the standard $\sqrt{g/L}$ for a point mass pendulum because of the rotational energy in the mass as the pendulum swings. Please use φ to denote the angle the string makes with the vertical and compute the adjusted angular frequency two ways:
    a. Use $T = T_{cm} + T_{rot}$ where T_{cm} is the kinetic energy of the center-of-mass motion and T_{rot} is the rotational kinetic energy to define the Lagrangian for the problem. Solve the equation of motion assuming a small oscillation amplitude. Determine the adjusted frequency.
    b. Is your result for the kinetic energy consistent with the parallel axis theorem?